Int. J. Solids Structures Vol. 16, pp. 545–562
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0020-7683/80/0601-0545/$02.00/0

PHENOMENA ASSOCIATED WITH THE CRUSHING OF METAL TUBES BETWEEN RIGID PLATES

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(*Received* 18 *April* 1979; *received for publication* 8 *September* 1979)

Abstract—Experiments are described in which various phenomena associated with the large deformation compression of metal tubes between rigid plates are examined. The observations from these tests led to the formulation of an alternative theory[10] for tube compression. The relevance of these observations and the resulting theory to other plasticity problems involving bending as the primary mode of deformation is discussed.

NOTATION

- A moment arm, as shown in Fig. 7(a)
- D diameter of the tube
- E Young's modulus
- E_p mean strain hardening modulus
- L length of the tube
- M_p, M_y fully plastic bending moment and bending moment at the onset of yielding of the section of a tube of unit length respectively
- P load acting on the tube
- P_1, P_2 loads corresponding to onset of plastic deformation and the establishment of full plasticity respectively under the load
- P_0 initial collapse load of a tube of unit length
- R radius of the tube
- a, b semi-width between the contact zones (with plates) of an unloaded tube at the ends and mid-section respectively, see Fig. 9.
- h anticlastic decay length at the ends of the tube
- t thickness of the tube wall
- δ deflection of the tube, defined as the reduction in the distance between the platens
- $\delta_1, \delta_2, \delta_0$ deflections at P_1, P_2, P_0 respectively
- σ stress
- σ_0 yield stress of the material in uniaxial tension
- ϵ strain
- θ angle, defining the position of the strain gauges, see Fig. 12(a)

INTRODUCTION

Large plastic deformations of engineering structures or structural components are of interest in several fields and one such is the design of metallic impact energy absorbing systems. This area of research has recently been reviewed by Johnson and Reid[1].

Rigorous whole field analysis of even the simplest structures incorporating realistic constitutive equations are rare and usually simplifying assumptions are made which lead to approximate expressions for the load-deflection relationship. In particular, when a bending mode of deformation predominates, it is customary to assume that the material is rigid-perfectly plastic and that plastic hinges are formed at critical sections of the structure as in the analysis of plastic collapse using the methods of limit analysis[2]. The load is allowed to increase beyond its collapse value and the geometry of the structure is modified by assuming that the rigid portions undergo finite rotations about the plastic hinges. This produces large deformation load-deflection relationships and permits a rough estimate of the energy absorbing capacity of the structure to be made[1].

A particular example of this approach is the analysis of the crushing of metal tubes between rigid plates. This will be discussed in detail below. A discrepancy exists between the results of the rigid-perfectly plastic theory and experiments which was attributed to the effects of strain hardening. An attempt was made to rectify this by allowing the bending moment at the hinges to increase as a function of the rotation about them but the load was still significantly underestimated, particularly at large deflections.

During the course of a recent investigation concerning the use of metal tubes in impact

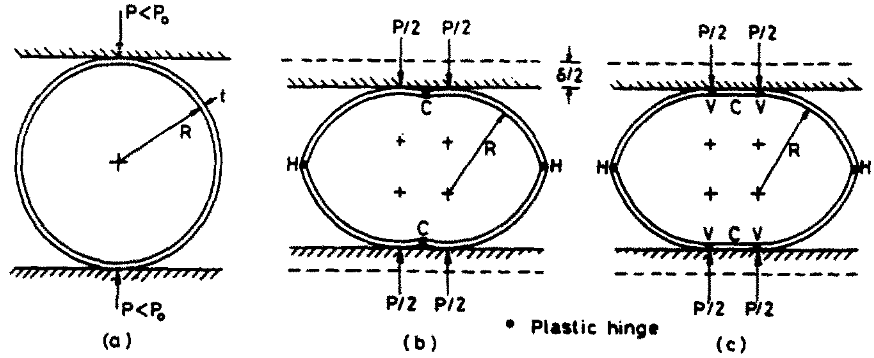


Fig. 1. Tube compressed between rigid plates: (a) Before collapse; (b) DeRuntz and Hodge mode of collapse; (c) Burton and Craig mode of collapse.

energy absorbing systems[3], a series of experiments was performed to examine the various features of the response of single tubes subjected to gross (approximately 80%) reductions of diameter produced by compression between rigid plates. The data so obtained highlight a number of phenomena which could be of interest both to mechanical and civil engineers, particularly with regard to the concept of plastic hinges, especially when large deformations are involved.

The elastic analysis of a thin ring subjected to diametrically opposed loads has been a topic of interest since the turn of the century[4]. The elastic behaviour of a tube can be deduced from that of the ring by using the well known relationship between the elastic constants for plane stress and plane strain deformations. The plastic collapse of a thin ring of elastic–perfectly plastic material subjected to diametrically opposed outward loads was first analysed by Hwang[5] and the same analysis is applicable to opposed inward loads. The post-collapse behaviour of a thin rigid–perfectly plastic tube compressed between rigid plates was analysed by DeRuntz and Hodge[6] and Burton and Craig[7] who proposed the modes of deformation shown in Fig. 1. The analyses for these two models produce identical post-collapse behaviour. Strain hardening was incorporated into the Burton and Craig model by Redwood[8] who assumed that the material was rigid–linearly strain hardening and was then able to include the increase in bending moment with rotation. However, the theoretical load-deflection characteristic still fell below the experimental one even with the modification for strain hardening, the discrepancy increasing with deflection.

In an attempt to discover the source of the remaining discrepancy between the theory and the experiments, the authors re-examined the problem experimentally. These experiments led to the establishment of a new large deflection theory, called herein the plastica theory, in which the side hinges were replaced by plastic regions whose behaviour can be modelled by standard elastica theory[9]. This model has been described in detail in a companion paper[10] and the results of its application to the tube compression problem are shown in Fig. 2. The aim of the present paper is to describe the experimental programme which led to the establishment of the plastica theory and to discuss those findings which may be generally relevant to other structural plasticity problems.

EXPERIMENTS AND OBSERVATIONS

Experimental procedure and the range of experiments performed

All the tests were quasi-static in nature and were carried out on an Instron Universal testing machine. The cross-head speed for all the experiments was 0.2 in. per min (0.5 mm per min). The specimens were made of seamless tubes, mainly of aluminium (HT30) and mild steel (CDS2) both in the as-received and annealed states. A few copper and brass specimens were also used in the as-received state. A limited number of tensile tests were conducted to determine the mechanical properties on specimens prepared from the tube stock and annealed under the same conditions as the tube specimens. In the main, however, the yield stress was estimated from the Vickers hardness of the material using the well-known relation between the two: yield stress at 8% strain is equal to a third of the Vickers hardness[11].

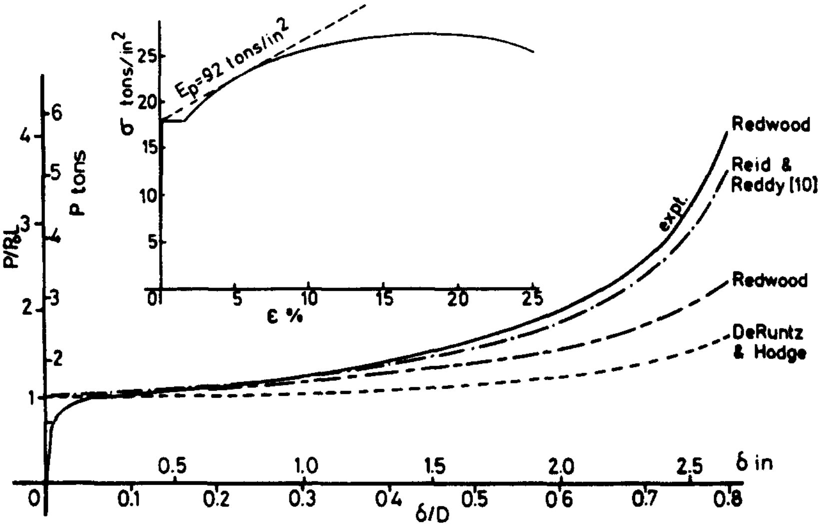


Fig. 2. Non-dimensional load-deflection plots; experiment and theory [8].

Non-dimensional load–deflection characteristics of tubes of some typical materials compressed between platens are presented in Fig. 3(a). Whilst this behaviour is broadly the same as that reported by previous workers, all the tubes do not show the same physical behaviour. In all tubes flattening to various degrees adjacent to the platens was observed before collapse. At larger deflections the central regions of these flats lifted off from the platens, beginning from the ends where the amount of separation was considerably more (of the order of a few thicknesses of the tube depending on deformation and material) than in the middle of the tube. When unloaded from a deflection of 0.8 times the diameter, the degree of separation in the central portion of the "contact" region decreased or disappeared completely leaving a flat, but that at the ends remained almost intact.

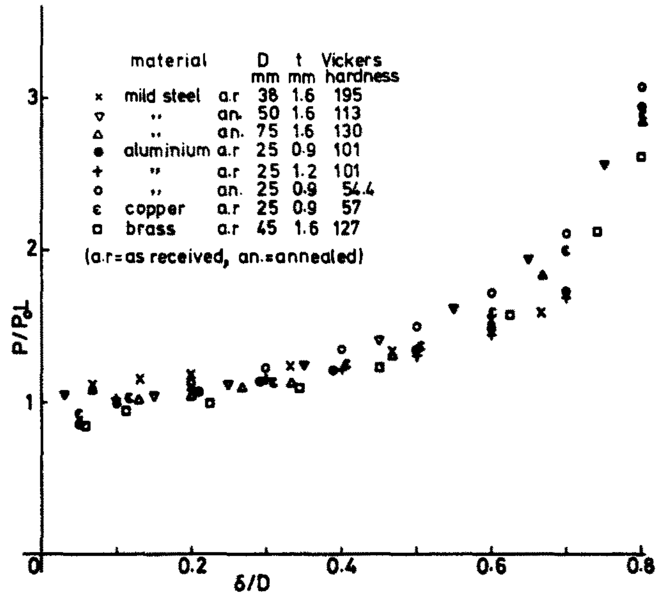
The general mode of plastic deformation ranged from that of a Burton and Craig mode in the annealed aluminium tubes to a DeRuntz and Hodge mode in a few as-received mild steel tubes. In these latter specimens small cracks were observed along the side generators. Some steel and brass tubes in the as-received state cracked completely along the sides. Annealed steel tubes behaved in an intermediate manner, the region in the vicinity of the generators at which initial contact with the platens occurred attaining an almost exact reversal of curvature at deflections in excess of $\delta/D = 0.3$ approximately. This curvature gradually reduced away from these central generators, passed through a zero value and eventually attained the initial curvature at the generators currently in contact with the platens. The end views of some typical tubes are shown in Fig. 3(b).

Data on a number of phenomena associated with the deformation of the tubes were collected from a series of separate experiments each of which is described below. The reasons for performing the experiments are briefly enumerated.

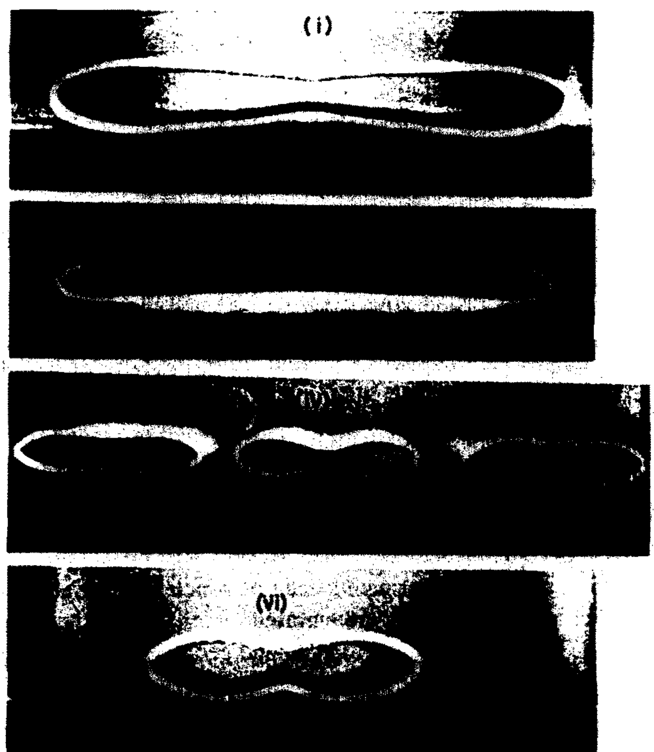
In order to assess the effects of strain hardening and geometry changes independently some intermittent annealing tests were conducted. The separation effects (which were at first thought to be contributing factors to the load-deflection behaviour) were studied more closely by replacing one platen with a glass plate so enabling the behaviour of the tube adjacent to the platen to be observed. The effect of the tube length on the load-deflection behaviour was examined and the extent of the end regions of the tubes affected by anticlastic curvatures was investigated. Strains were measured both at the sides and in the region adjacent to the platens and a brief study of the effects of friction between the plate and tube was also made.

Tests on annealed aluminium tubes

(a) *Tests on as-received and annealed tubes.* As-received tube specimens and several identical specimens of the same stock of aluminium ($L = 75$ mm, $D = 25$ mm, $t = 0.9$ mm) which were annealed—soaked at 360°C for half an hour and cooled in the furnace—were tested.



(a)



(i) annealed MS: D=75, t=1.6 (ii) annealed Al: D=75, t=1.6
(iii) as received Al (iv) intermittently annealed Al
(v) as received Cu: [D=25 t=0.9] and
(vi) as received MS: D=50, t=1.6

(b)

Fig. 3. (a) Experimental load-deflection data for different materials; (b) End views of tubes compressed between platens.

Tension specimens prepared from strips of the same tube stock were tested to deduce the mechanical properties. The hardnesses of the as-received and annealed specimens were 101 kgf/mm^2 and 54.4 kgf/mm^2 respectively. The limit of proportionality of the material, found from tension tests, was 114 MPa and the 0.2% offset yield stress was 124 MPa.

The results for the set of annealed test specimens were virtually identical, as were those of the as-received specimens. Typical load-deflection plots are shown in Fig. 4. The two curves are parallel to each other in the region of plastic deformation except in the final stages. The modes of deformation for the two cases were identical. Flattening occurred at the platens at the beginning of plastic deformation, followed by lifting-off from the platens within these flat regions as the deformation continued. The plastic separation was more at the ends of the tube than at the centre.

(b) *Interrupted annealed tests.* To remove the effect of strain hardening three more annealed tube specimens, identical to the previous ones, were compressed by 2.5 mm, annealed again and then compressed by a further 2.5 mm and the process repeated. There were only minor differences between the results for the three specimens. The load-deflection plot for the interrupted annealed tests is also shown in Fig. 4, where the theoretical curve for a rigid–perfectly plastic material is also shown. During deformation, these interrupted annealed test specimens showed a greater tendency for separation though the mode shape was still not that of the DeRuntz and Hodge model.

(c) *Observations.* From an inspection of the load-deflection plots it is apparent that strain hardening has the main effect on the differences between the rigid-perfectly plastic theory and the experiments. The experimental results of the interrupted annealed tests agree excellently with the theory but those of the once annealed tubes (before the tests) show a large discrepancy. This discrepancy has been noted previously[12]. The load for the once-annealed specimen at a deflection of $0.7D$ is approximately twice the theoretical value. The effect of strain hardening is also apparent in the results of the as-received and the annealed specimens. At a deflection of about $0.1D$ the as-received tubes are more than twice as strong as the annealed ones, the ratio of collapse loads being the same as that of their hardnesses. However, the annealed tubes have a greater capacity to strain harden and get stronger with increasing deformation at a higher rate than the as-received tubes. These latter have been initially cold worked during manufacture and hence strain harden to a lesser extent. The ratio of loads at deformation of $0.8D$ and $0.1D$ for the as-received and annealed tubes is 1.8 and 2.7 respectively.

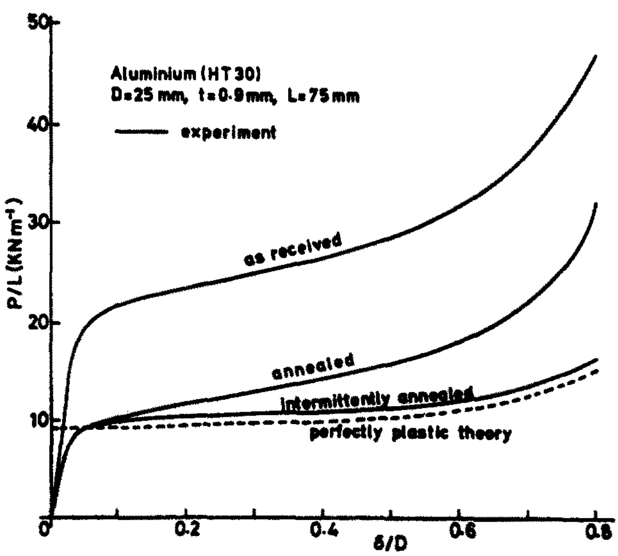


Fig. 4. Load-deflection characteristics of as-received, annealed and intermittently annealed aluminium tubes.

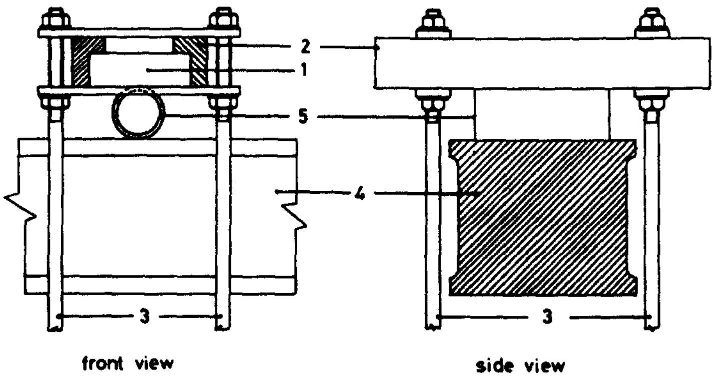


Fig. 5. Glass platen apparatus.

Glass platen tests

(a) *Apparatus.* The general arrangement of the experimental apparatus is shown in Fig. 5. A thick glass block (25 mm thick and 75 mm wide) (1) was held in an aluminium frame (2) and this unit was fixed to the bed of the Instron testing machine by four rods (3), two on either side of the cross-head (4), so that the glass block straddled the cross-head. The specimen (5) was placed in between the cross-head and the glass block. With this arrangement the usual unloading (upward) motion of the cross-head compressed the specimen. A dial gauge was used to measure the relative motion between the cross-head and the glass block, which is equal to the deformation of the specimen. The contact between the tube specimen and the glass platen was observed from above and it was possible to photograph this area. A thin smear of oil on the lower side of the glass (in contact with the specimen) helped to produce a very clear view of the contact area(s). It was not possible to include the load cell to measure the load with this simple set up but this was thought to be unnecessary as the load-deflection behaviour of these tubes was well-known from previous tests.

(b) *Observations.* A sequence of photographs of the specimen taken with this set-up at different deformation levels is shown in Fig. 6. As one might expect the line of contact grows to

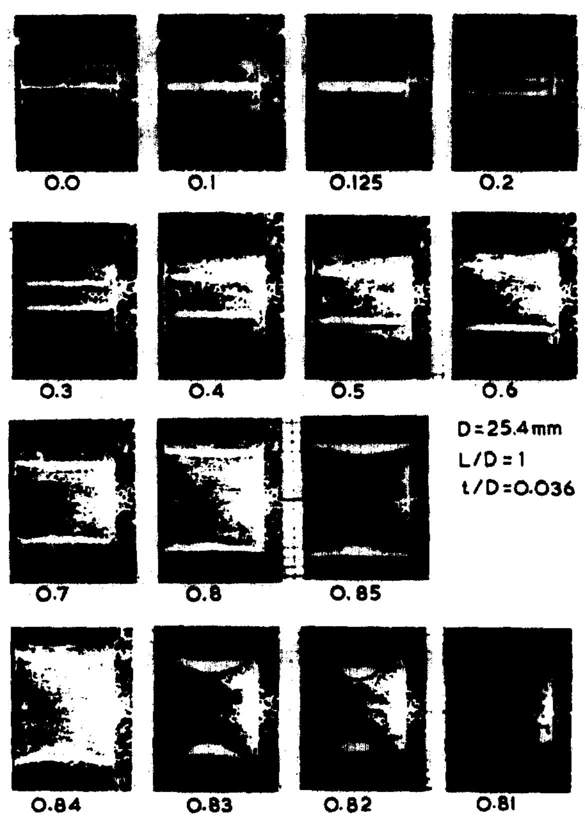


Fig. 6. Development of contact zones with eformation.

a rectangular area of contact with the width of the rectangle getting wider as the deformation progresses. Contact at the ends of the tube is lost early in the loading process rounding the corners of the rectangle near to the edges of the tube. This is due to anticlastic curvature effects. This happens when the tube is still elastic and persists into the plastic range. Also, it can be seen that this rectangular area splits, along the line of initial contact, into two bands quite early in the deformation history, at $\delta/D = 0.125$ ($\delta/t \simeq 3.5$), and the two bands move away from each other as the deflection increases. However, this initial separation was found to be elastic and permanent (plastic) separation was observed only after the deflection was increased to above 0.3 times the diameter.

The bands of contact are neither rectangular nor symmetrical in themselves though they are symmetrically placed with reference to the centre line, i.e. initially the top generator. Whilst the outsides of these bands are straight, they are curved up to a certain length from the ends on the inside, the extent to which this curve extends increasing with deformation. This curvature is an indication of the non-uniformity of lift-off from the platen in the region between the contact bands and an indication of the anti-elastic decay length.

The bands of contact separated from the platen at the ends of the tube early in the deformation history and regained contact only in the later stages of deformation. The movement of the contact zones during unloading is shown in the last four photographs of Fig. 6. Contact is lost at the ends, the contact bands grow wider and move inwards towards each other indicating a larger elastic recovery in the central region than at the ends.

(c) *Quantitative results and discussion.* The central separation between the contact bands was measured from the photographs and its variation with deflection is shown in non-dimensional form in Fig. 7(a). The variations predicted by the theoretical models due to DeRuntz and Hodge ($2a = \delta$), Burton and Craig ($2a = D\sin^{-1}(\delta/D)$ and the plastica theory are also shown in this figure. The total width of the tube, D_H, was measured and the magnitude $A = (D_H - 2a)/2$ was calculated. The quantity A is the distance between the two equal and opposite forces on the curved portion of a quadrant of the tube (see inset in Fig. 7a), and hence is the moment arm responsible for the maximum moments caused by these forces. It can easily be shown that

$$A = \frac{D}{2}\left[1 - \left(\frac{\delta}{D}\right)^2\right]^{1/2} \tag{1}$$

for each of the rigid-plastic theoretical models. The experimental and theoretical variations of A/D are also shown in Fig. 7(a). The difference between the rigid-plastic and experimental curves is an indication of the difference between the actual geometry of a deforming tube and that computed by the earlier models. The influence of geometry is highlighted in Fig. 7(b) in which a comparison is shown between the experimental load-deflection plot, the theoretical prediction of the earlier models, the plastica theory and the estimate $P = 4M_p/A$ for the applied load given by the experimental moment arm A, assuming that the magnitude of the plastic moment at H and V (Fig. 1) is M_p (i.e. neglecting strain hardening in the hinges). The consideration of strain hardening (via the plastica theory) can be seen to improve the agreement between theory and experiment in both parts of Fig. 7, although the theoretical value of A is slightly less than the observed value (Fig. 7a).

The inaccuracy in the prediction of the moment arm produced by the plastic hinge models stems presumably from the fact that the analysis permits a discontinuity of slope at H, which does not in fact occur. The plastica analysis replaces the hinge at H with a plastic zone whose extent is determined by the mean strain hardening modulus of the material and the tube geometry. Furthermore, the condition is imposed that the tangent to the central surface of the tube is vertical at H.

Effect of tube length

It was observed that the separation from the platens always started from the ends of the tube and this separation at the ends was always permanent, i.e. plastic, irrespective of whether that in the middle was plastic or elastic. It was therefore not difficult to conclude that short tubes (rings) might exhibit the separation phenomenon earlier because of the overlapping of the zones in which anticlastic effects are apparent. Consequently the load-deflection characteristics

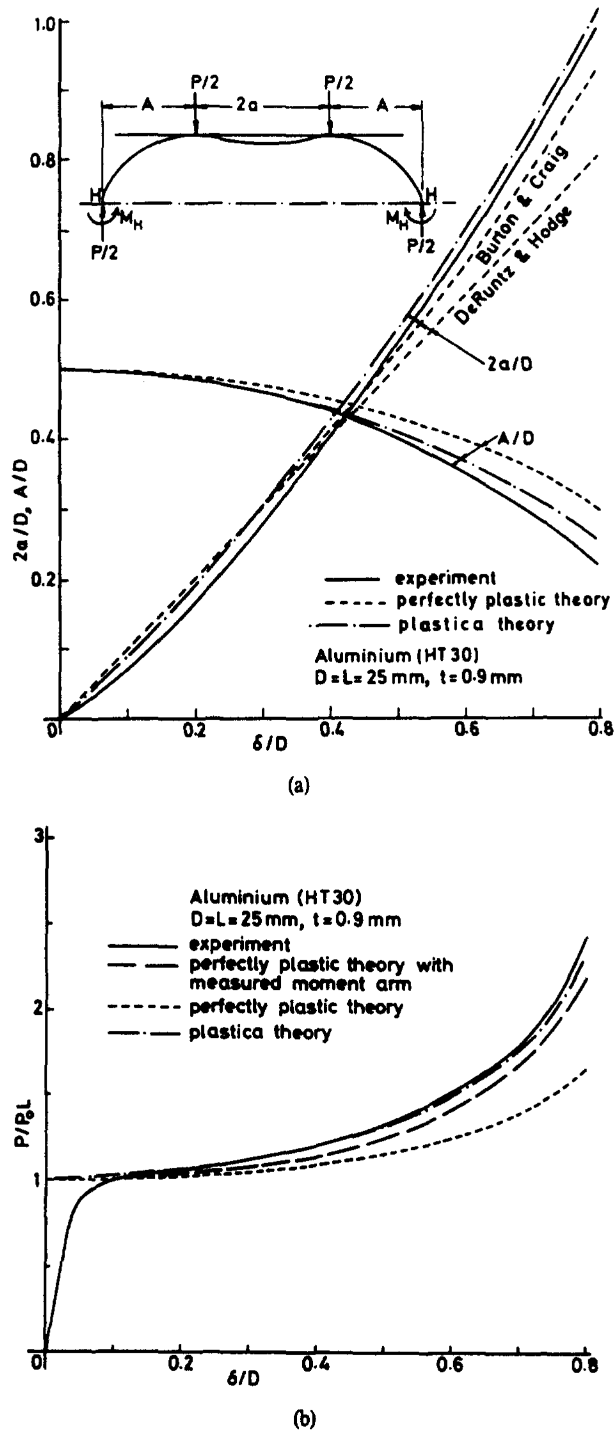


Fig. 7. (a) Flat width and lever arm measurements from glass platen tests; (b) Experimental, empirical and theoretical load-deflection characteristics.

of rings might be expected to be different from those of tubes. A series of tests were conducted on the as-received aluminium tubes ($D = 25$ mm, $t = 0.9$ mm) of lengths varying from 3 to 200 mm.

Whilst in all the specimens the ends appeared to lift off permanently from the platens at a deflection of ~ 3 mm, such separation along the whole length occurred earlier in rings than in tubes. In the smallest ring ($L = 3$ mm) total permanent separation was seen to occur at a deflection of about 4 mm whereas such separation of all tubes longer than 12.5 mm occurred at a deflection of about 8 mm.

The load-deflection behaviour of each of the specimens is shown in Fig. 8. The difference in the initial collapse load between rings was found to be no more than the difference between plane stress and plane strain behaviour, tubes being about 12% stronger than the rings. (If σ_0 is the effective yield stress in plane stress, that in plane strain will be 1.55 σ_0). But as the deformation increased the difference decreased and gradually the rings became stronger (carrying a load per unit length 25% more than the tube at a deflection ratio of 0.8). This should be partly due to more plastic work consumed per unit length for the anticlastic deformation (axial bending) of the overlapping end zones and partly due to the resulting geometry changes. At large deflections it was found from glass platen tests that the distance between contact zones was slightly larger in rings than in tubes which would result in a correspondingly smaller moment arm.

A simple attempt to assess the contribution of the anticlastic curvatures in resisting the applied load has been made[3] making use of the anticlastic decay length measurements described below.

Anticlastic effects

All the tubes showed anticlastic bending deformation in the end regions to a greater or lesser extent. The loci of the highest points on transverse sections along the length of a tube in an unloaded state are shown schematically in Fig. 9, (a) for tubes which separate plastically from the platens, (b) for tubes which do not show such plastic separation, and (c) for short tubes in which the end effects overlap. The latter showed permanent separation irrespective of whether the longer tubes of the same material did or did not. The distance between these loci at the ends of a tube from which the load has been released was nearly equal to that in the loaded state for all cases. In case (a) the separation at the ends, $2a$, was, in general, greater than that at the middle, $2b$. In annealed mild steel and intermittently annealed aluminium the two were nearly equal and the loci were almost straight lines. In (b) the ends had lifted off permanently from the platens but the middle region did not. The anticlastic effect decayed over a distance, h, as shown in Fig. 9, which is approximately equal to $2a$ for most tubes of case (a) and a for case (b). The boundary of these decay zones was a continuous curve, resembling a semicircle in case (b).

To study the variation of this decay length with deflection, it was measured at different deflections in tubes belonging to both types (a) and (b). A straight edge was held against the tube along the centre-line of the contact zone and the distance from the edge of the tube to the

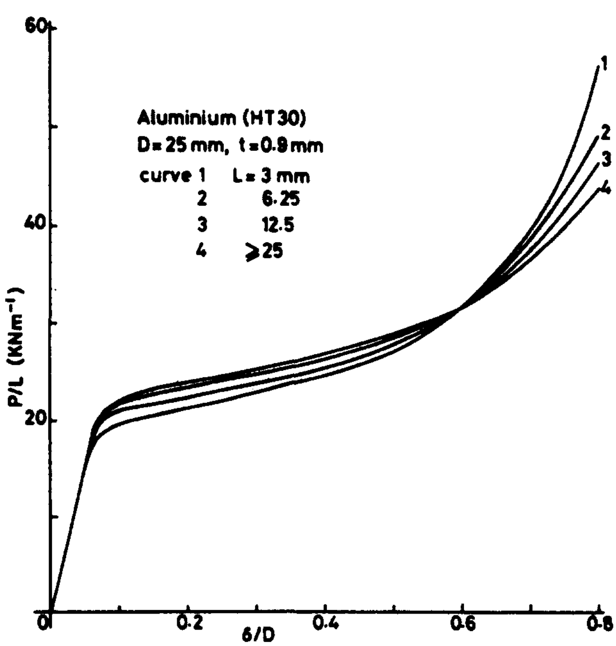


Fig. 8. Effects of tube length on the load-deflection behaviour.

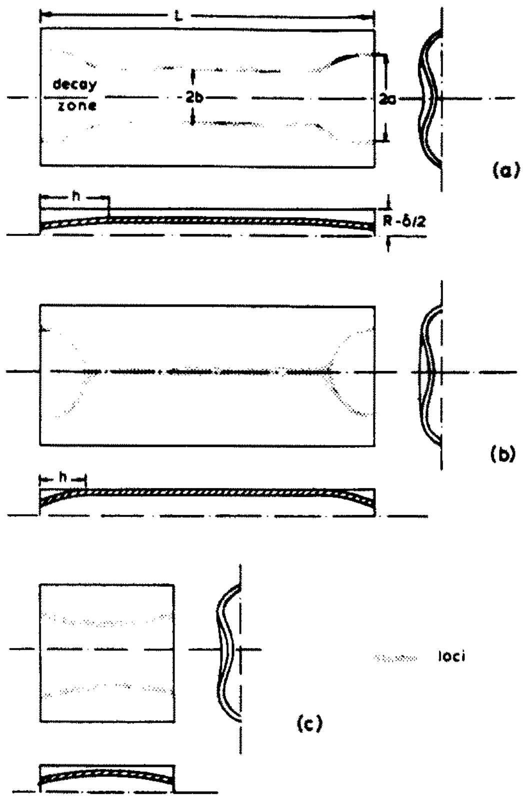


Fig. 9. Loci of highest points in the longitudinal cross sections of unloaded tubes; (a) for tubes showing permanent separation; (b) for tubes which do not show permanent separation; (c) for short tubes.

point of contact between the tube and the straight edge was measured as the decay length h. There was considerable variation in the four measured values of h in a tube at a given deflection. A few more accurate measurements were made using a Ferranti 3-D measuring machine, with an accuracy of 0.001 mm. However, these showed similar variations and the crude measurements using a rule were always within the variation bands of these more accurate values. It was therefore considered that the quick and easy measurements with the rule would suffice to detect the general trend of variations in h.

The decay lengths, at any given deformation, of annealed tubes which showed permanent separation from the platens were not significantly different from that of the as-received ones, considering the variations in a given tube. However, the values of h taken from the interrupted annealed tests were less than for other tubes of case (a) and lay within 15% of the value of a. This was also true for tubes of case (b). The variation of these measured decay lengths with deflection are shown in Fig. 10. The variation of the ratio $2a/D$, which was the same for both cases (a) and (b), is also shown in this figure.

Strain measurements

The phenomenon of separation from the platen of the flattened tube adjacent to it during deformation cannot be explained by any of the existing models. The curvature of this region, though not measured, appeared to vary along the width. With the object of getting more information about this region, surface strains were measured using electrical resistance (post yield) strain gauges. Strains at the side hinges were also measured at the same time to provide data for comparison with the plastica theory.

(a) *Specimens.* Annealed mild steel (soaked at 900°C for 20 min and air-cooled) and aluminium (soaked at 360°C for 30 min and furnace cooled) tube specimens of identical dimensions ($D = 75$ mm, $t = 1.6$ mm and $L = 150$ mm) were used. These were chosen because the former showed a completely reversed curvature at top and bottom and the latter did not show any significant plastic separation at all. The yield stress of the two materials measured

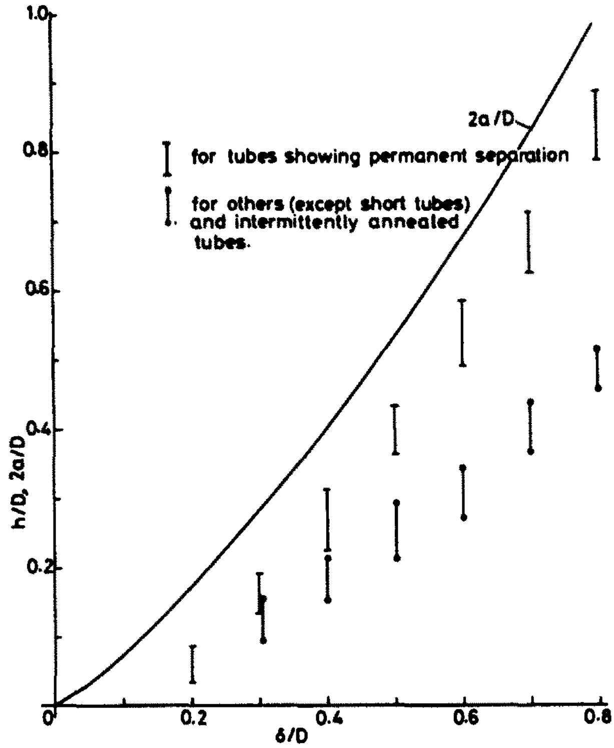


Fig. 10. Decay lengths at the ends of the tubes.

from prepared tensile specimens annealed with the tube specimens were 293 MPa and 103 MPa respectively. The nominal stress-strain diagrams of these materials are shown in Fig. 11.

(b) *Gauge locations.* All the gauges were mounted with their axes in the circumferential direction of the tube and their locations on the mild steel specimen are shown in Fig. 12(a). Gauge 3 was offset axially to avoid overlapping with gauges 2 and 6 (for which reasons gauges 4

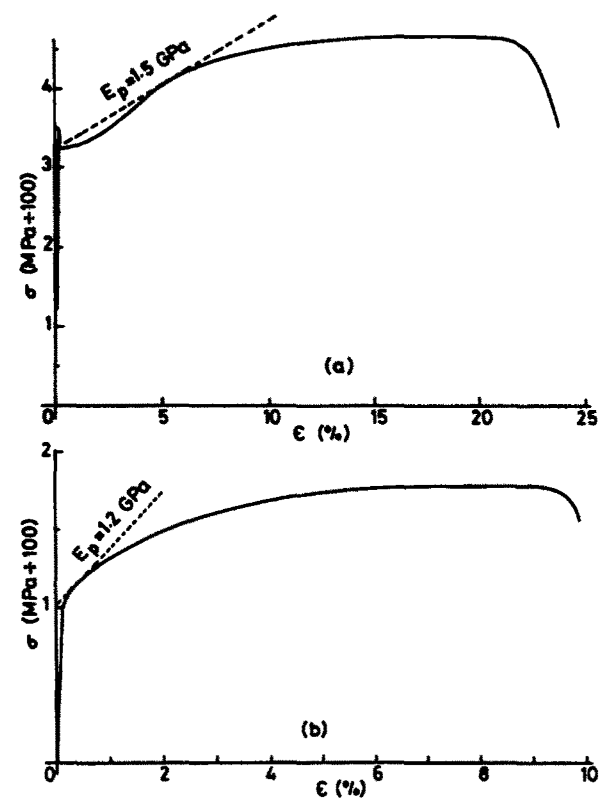


Fig. 11. Nominal stress-strain diagrams for mild steel and aluminium tube material used in strain measurements.

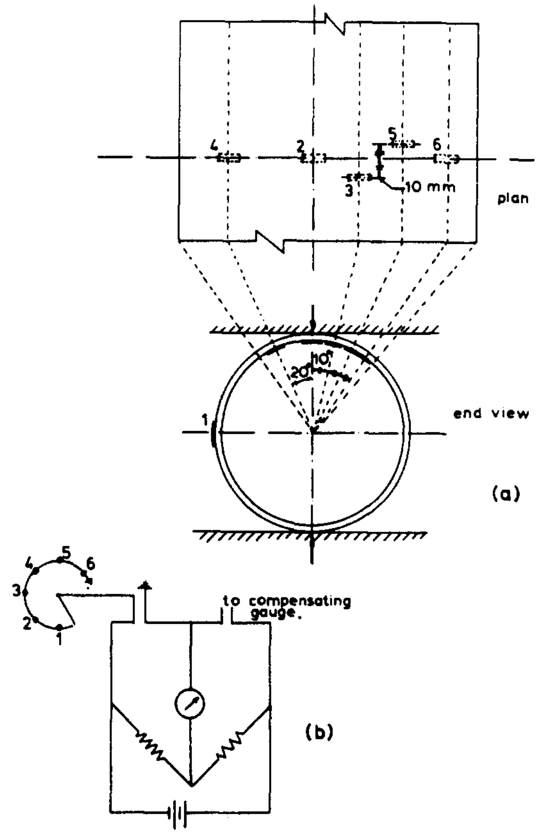


Fig. 12. (a) Gauge locations on the mild steel specimen; (b) Schematic of the strain gauge bridge formation.

and 6 are on opposite sides of 2) and gauge 5 was mounted in the position shown to detect any differences of strains in this offset section from those at the central section (by comparison with strains in gauge 4).

The connections of the gauges are shown schematically in Fig. 12(b). Each of these gauges were, in turn, made to form the active arm of a two arm bridge by using a switching box, the other arm being an identical gauge mounted on a similarly prepared tube for the purpose of temperature compensation. This two arm bridge was internally completed, excited and monitored by a deflection type bridge amplifier capable of measuring strains from 10^{-5} to 10^{-1}.

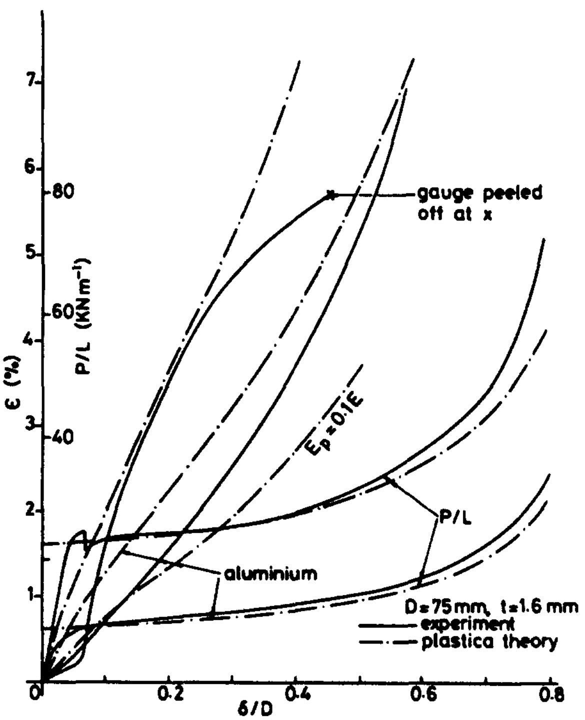
In the aluminium specimen only two gauges were mounted at locations 1 and 2 in Fig. 12(a).

(c) *Results.* Strains were measured at different levels of deflection during the loading cycle. When the strains in the different gauges were read the deflection, and hence the strain, was held constant and resulted in a drop in load due to stress relaxation. The drop was up to 6% on occasions in the case of the mild steel specimen and the load increased very steeply, almost instantaneoulsy, when the loading was restarted. In the aluminium specimen the drop in load was not so high but the time spent at each deflection was also less because only two gauges were to be monitored.

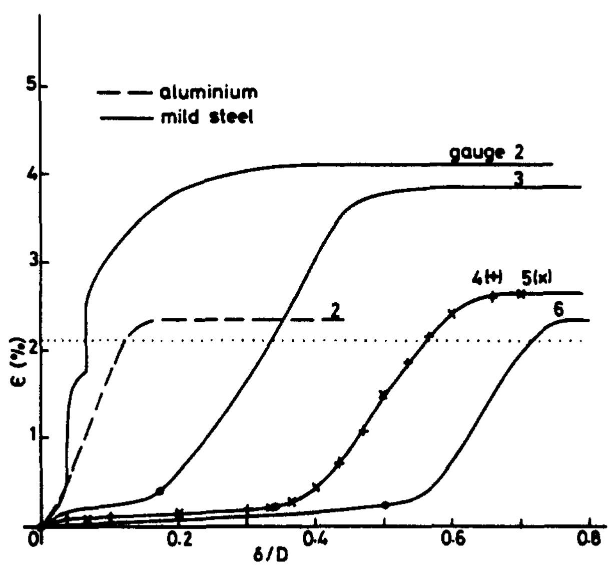
The strains measured at the location 1 and the load-deflection behaviour of the two specimens are shown in Fig. 13(a) in which the corresponding results of the plastica theory are also shown. Strains at locations 2–6 (only 2 in aluminium) are plotted in Fig. 13(b) against deflection. All these strains are tensile.

(d) *Comparison between measured and predicted strains.* The load P_1 and the deflection δ_1 at the onset of plastic deformation along the generator under the load, i.e. when stress in the outer fibres is σ_0, the corresponding values P_2, δ_2 when this section becomes fully plastic and P_0, δ_0 at collapse can be computed from equations given in Ref. [5]. Table 1 compares these theoretical values with the results of the present experiments on the mild steel specimen and also includes surface strains under the load at P_2 and at the sides at load P_0.

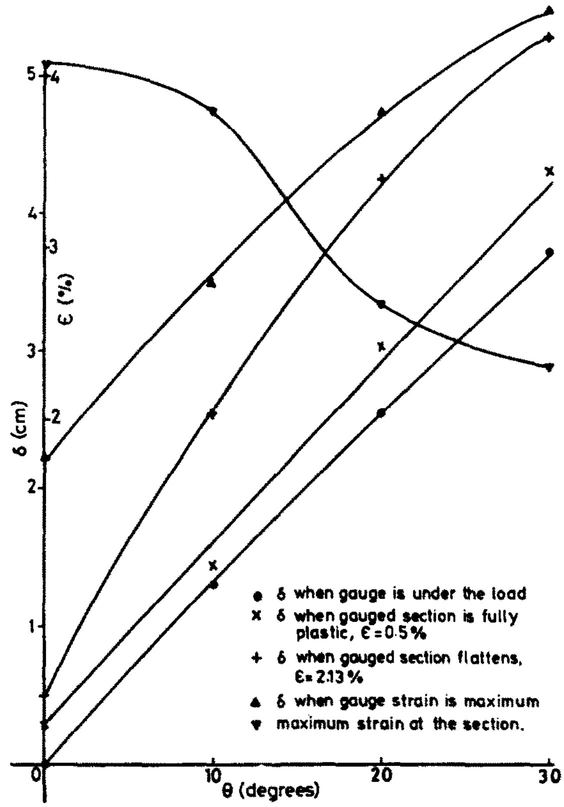
The agreement between the theory and the experiment for loads and deflections up to the


gauge peeled
off at x
P/L
aluminium
D=75mm, t=1.6 mm
experiment
plastica theory
$E_p = 0.1E$
ε (%)
P/L (KNm^{-1})
δ/D

(a)


aluminium
mild steel
gauge 2
3
2
4(+) 5(x)
6
ε (%)
δ/D

(b)



(c)

Fig. 13. (a) Strains at locations 1 and load-deflection characteristics; (b) Strains at locations 2–6 (only 2 for aluminium); (c) Compilation of strain gauge data.

Table 1. Comparison between experiment and theory [5]

Parameter	Theory [5]		Experiment Fig. 13 (a)
	equation	value	
δ_1 at $P_1 = \frac{\pi M_y}{R}$, $M_y = \sigma_0 t^2/6$	$\frac{0{\cdot}149 P_1 R^3}{EI}$	1·27 mm	1·5 mm
δ_2 at $P_2 = \frac{\pi M_p}{R}$, $M_p = \sigma_0 t^2/4$	$\frac{0{\cdot}149 P_2 R^3}{EI}$	1·9 mm	2·2 mm
P_0	$\frac{4M_p}{R}$	512 N	520 N
δ_0	$\frac{1{\cdot}142 P_0 R^3}{EI}$	4·3 mm	5 mm
Strain at P_1	$\frac{\sigma_0}{E}$	0·13%	0·12%
Strain at P_2 (gauge 2)			0·4%
Strain at P_0 (gauge 1)			0·5%

collapse load and the strain at load P_1 is good. Consequently, from the experiment the strain at the side hinges at the collapse load can be reasonably taken as 0.5%.

Similar agreement can also be found in the aluminium specimen in the pre-collapse region. However, the deflection and load at collapse cannot be defined as precisely.

The agreement between the measured strains and those predicted by the plastica theory in the plastic deformation range do not seem to agree well except in a limited range of deformation for mild steel and at larger deformations for aluminiun specimens. The reasons for these will be discussed below.

(e) *Some observations.* In the mild steel specimen at the collapse load there is a sudden jump in the strain at location 2 crossing the value corresponding to zero curvature, i.e. 2.13% for the present specimen. The upper level of this strain discontinuity, about 2.4% is nearly the same as the strain needed for the stress to regain a value equal to the upper yield limit in the tensile test, Fig. 11(a). After collapse, while the strain at the side hinges (location 1) continues to increase monotonically, that in the gauge 2 settles to a constant value corresponding to a curvature change of $2/R$, i.e. the tube acquires the same curvature but in the opposite sense at this section. In the aluminium specimen no jumps of strain occur and the region adjacent to the platens only flattens as indicated by the measured strains and the appearance of the specimen after the test. At any given deflection the strains in the aluminium specimen, at two points considered, are less than those in the mild steel specimen.

The strain when a section becomes fully plastic, as noted from the strain at the side hinge, is approximately 0.5% in mild steel. Strains in the sections slightly offset axially from the mid-section are the same as seen from the outputs of gauges 4 and 5 and so the output of gauge 3 can reasonably be considered to be the same as that at a position of 10° from the vertical in the mid-section. In Fig. 13(b) the points at which the longitudinal sections through gauges 2, 3, 4 and 6 were directly under the edge of the contact zone are marked by bold circles on the corresponding curves. It can be seen that the inner surface strains at a given section have not reached 0.5%, when the top fibre of this section is in contact with the platen. The yield point is reached after the contact line has moved out, by distance of 1.5–3t, from the strain gauged section. The strain corresponding to the flattening of the tube (zero curvature) at a section is reached after the section has moved well into the separated region, about 12–17 mm or 7.5–10t. It is interesting to observe that the maximum circumferential strain decreases as we move away from the central position outwards towards the stationary hinge. It appears that this maximum value may not be sufficient to cause the flattening of the tube beyond an interval of 15° past strain gauge 6. These results are summarised in Fig. 13(c).

Figure 13(b) indicates that the magnitude of the bending moment in the tube wall (as reflected by the strain level) under the lines of contact with the platens is less than that associated with complete flattening and is of the order of M_p, the initial fully plastic moment. This, together with the observation that the mean change in curvature within the "contact" region (i.e. between the contact lines) is approximately $1/R$, led to the assumption in the plastica analysis that the contact region was flat and terminated at the contact lines with two travelling plastic hinges at which the bending moment was M_p. This degree of simplification was very useful in the plastica analysis and allowed the authors to avoid a detailed discussion of the interaction between a plastic bending region and a contact stress field. This remains an interesting problem in its own right and has appeared in the literature with regard to stationary plastic hinges vis à vis the difference between the fully plastic moments for beams as measured in three and four point bending tests (see p. 328, Ref. [2]).

Effect of friction. As a tube compressed between platens deforms, the surface of the tube not initially in contact with the platen comes into contact with it and again moves out of contact as the deformation continues. Due to this relative motion friction will be present between the two surfaces and Redwood[8] proposed that this might account for the discrepancy between his theory and experiment. To assess the effect of friction on the load-deformation behaviour of tubes, compression tests were conducted on identical tube specimens using smooth, well-lubricated platens and comparatively rough unlubricated platens. No noticeable differences were present in the load-deflection curves and the two specimens looked alike after being deformed to the same extent.

The bottom platen was replaced by two plates which were set on rollers. No movement of these plates was seen during deformation of the specimen. Half-way through this test the specimen was unloaded, the plates were separated slightly and the test was continued. The two plates moved towards each other. However, it was possible to restrain this motion easily by hand. It was decided, therefore, not to pursue friction force measurements as these forced can only be a few tens of Newtons and the effect of these forces on the overall behaviour was clearly not appreciable.

DISCUSSION

A first glance at Fig. 3(a) gives the impression that as-received and annealed tubes of the same and of different materials behave in a similar manner. Whilst this is broadly true, a closer examination reveals that data for annealed tubes is at the lower end of the small scatter band at smaller deflections and moves towards the higher end at larger deformations. This effect, which would appear to be related to strain hardening, can be seen more clearly in Fig. 4 from the load-deflection curves of as-received and annealed tubes. The results of the interrupted annealed tests shown in this figure indicate that the major source of disagreement between the rigid–perfectly plastic theory and the experiments is indeed the neglect of strain hardening. When the material is made to yield at the same stress by annihilating the effects of strain hardening through intermittent annealing during deformation the behaviour of the tube can be represented accurately by the perfectly-plastic solution. Plastica theory also indicates a steepening characteristic for materials with smaller σ_0/E_p values (for any given R/t ratio) which is consistent with the above observations.

The behaviour of the tube adjacent to the plate, i.e. separation or otherwise, does not appear to influence the load-deflection characteristic significantly. The interrupted annealed test specimens showed a greater degree of separation than the others and yet the results still agree reasonably well with the rigid–perfectly plastic theory. The difference between the perfectly-plastic theory and the interrupted annealed test results are most likely due to anticlastic curvature effects at the ends of the tube. The separation phenomenon would seem to be related to the contact stress-plastic bending interaction discussed above and an explanation for it depends upon a clear understanding of this. (That this effect may be of practical significance is supported by the observation that separation within the contact zones is a noticeable feature of certain hollow metal ring seals[13]).

The glass platen tests revealed that the moment arm is smaller than the theoretically predicted values thus requiring larger applied loads to produce the same moments. The load-deflection characteristic computed for a rigid–perfectly plastic material with the help of this empirical moment arm (Fig. 7b) shows a far better agreement with the experimental results than the theory. The differences that still exist should be accounted for by increases in the fully plastic moment arising from strain hardening and other, secondary effects. This difference in geometry should also be due to strain hardening as can again be inferred from the agreement between the interrupted annealed tests and the simple theory. The combined effect of the changes in stress–strain relations and the geometry of the tube due to strain hardening would, therefore, seem to be responsible for the large differences between the simple rigid–perfectly plastic theory and the experiments. This is borne out by the results of the plastica theory given in Figs. 2 and 7.

Rings are always stiffer (i.e. have a steeper slope in the P/L vs δ/D diagram) than tubes in the plastic region, presumably due to the proportionately greater effect of forces associated with the production of anticlastic curvature. Collapse at a lower load is due to the difference between the yield stress under plane stress and plane strain conditions.

Strains measured on the specimens agree well with those predicted by the analytical expressions due to Hwang[5] in the early stages of deformation. For mild steel tubes collapse is well defined and taking an outer fibre strain of 0.5% to correspond to the section becoming fully plastic would seem to be reasonable from the experimental results. Such a value cannot be identified with equal certainty for the aluminium tube as the collapse load itself cannot be defined as clearly. However, if a load at the beginning of the plateau of the load-deflection curve is assigned as the collapse load then the corresponding strain, taken from the experimental results, would be 0.65% approximately.

The strains predicted by the plastica theory seem to agree well only in the range of deformation $\delta/D = 0.1 - 0.35$ for mild steel. A disagreement is to be expected when $\delta/D < 0.1$ because the strain gradients are very large in the vicinity of H during initial yielding, enhanced by the discontinuous yielding nature of mild steel and the fact that the strain gauge only measures an average strain over the gauge length. The gauge peeled off at $\delta/D = 0.45$ and possibly started slipping earlier than this, which might account for the increasing differences after $\delta/D \sim 0.3$. The disagreement in the case of aluminium is also partly due to the averaging effect mentioned above. However, the major discrepancy here appears to be due to using too small a value for the hardening modulus, E_p, during the initial stages of deformation. The value of E_p used here is $\sim 0.016E$ (E is the elastic modulus) whilst E_p will be of the same order of magnitude as E at the beginning of plastic deformation and gradually decreases with deformation. A curve for predicted strain with $E_p \sim 0.1E$ ($= 7.6$ GPa) is shown in Fig. 13(a) and agrees well with the experiment during the early stages of deformation, though it falls short of the measured strains later. So it appears that a more realistic modelling of the stress–strain curve is necessary in order to achieve a better agreement over the entire range of deflection.

The tube adjacent to the platens becomes flat at (for mild steel) or after (for aluminium) the collapse load, though the section has become fully plastic well before this load is reached. The sudden jump in the strain at this symmetrical section in the mild steel specimen at the collapse load is probably related to the upper and lower yield point phenomenon and the attainment of a bending moment corresponding to the upper yield point at the side hinges. This discontinuity would also seem to be responsible for the plastic separation and subsequent complete reversal of curvature in this region. The increase of strain in aluminium is gradual and should be representative of copper, brass, etc.

With regard to the section under the contact zone, it would appear that this section does not become fully plastic when under load due to the suppression of plastic deformation at these sections produced by the influence of the contact stress field. The plastic hinges trail (i.e. lie inside) the contact zones by a distance of about 1.5–3 thicknesses and this trailing distance appears to increase slightly with increasing deformation (and hence increasing load and consequently increasing contact stresses). The degree of plastic deformation varies in this region, being more at the centre and reducing to zero at the contact zone, the maximum being such that the change in curvature is not more than $2/R$, twice the initial curvature. Thus the calculation of plastic work needed to deform this region requires more accurate information about the variation of curvature over this area. However, the average curvature might well be that corresponding to flattening, i.e. $1/R$, which renders the simple assumption of flattening in the "contact" region, as in the Burton and Craig model, adequate for the purpose of producing a theoretical model which is adequate for predicting overall load-deflection behaviour.

Of course, in the above it is assumed that bending strains dominate those measured. It is a simple task to check that the compressive strains due to the normal stress resultants are negligible. In the gauge at the 30° position, for example, the in-plane compressive force on the section at the collapse load is about $150\,N$ and hence the compressive membrane strain is 18×10^{-6}. Similarly at the side gauges at a deflection ratio of 0.6 the compressive strains can be found to be 56×10^{-6} which is negligible compared with the strains at these sections which are of the order of 10^{-1}.

The mode of deformation of a tube compressed between platens depends on the material, its state and also dimensions. Pre-worked mild steel specimens which show a rigid–perfectly plastic behaviour (or materials which are strain softening) deform in a four stationary hinge (DeRuntz and Hodge) mode. Strain hardening materials behave differently. Rings and short tubes always show permanent (plastic) separation from the platens. Thin tubes of materials with greater strain hardening capacity do not show such permanent separation, though they will exhibit elastic separation, except at the ends which are affected by anticlastic effects.

CONCLUSIONS

The use of rigid–perfect plasticity and the concept of plastic hinges has proved to be extremely helpful in determining the loads at which collapse or the onset of significant plastic deformation occurs. This is particularly so when applied to mild steel for which localised plastic deformation occurs as a result of the upper and lower yield point phenomenon. However, if one wishes to analyse the large deformations of simple structures and structural elements which deform in modes principally involving bending, then the consequences of strain hardening need to be considered. These involve geometric variations in the zone over which plastic deformation occurs as well as increase in the fully plastic bending moment of the section due to strain hardening. Both these effects are found in the crushing of tubes between rigid plates and the discussion given above is supplemented by that given in [10].

The comparison between measured and predicted strains in the vicinity of the side hinges indicates that more detailed agreement requires the use of more refined constitutive equations than the linear strain hardening one on which the plastica theory is based. This is particularly so if the data from tube compression tests are to be used to determine a range of material properties as discussed in [14].

Whilst the effects of strain hardening constitute the main phenomena responsible for the discrepancy between earlier theories of tube crushing and experimental observation, a number of other aspects of the problem have been identified which seem to be relatively untouched in the literature. Principal among these are the effects of anticlastic curvature, most significant for short tubes and rings, and the interaction between contact stresses and plastic bending. This latter is particularly interesting in the present problem where one has an increasing load following a plastic hinge which is travelling through the tube wall. As stated above, the separation phenomenon would seem to be closely related to this. Each of these topics would benefit from a separate study.

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